Template for summary of Tuning subject area findings

Mathematics

Introduction to the subject area

Mathematics, because of its abstract nature, is applicable to almost any discipline, since it identifies patterns that are common to many different areas. As a discipline its roots can be traced back through all the major civilisations to the earliest recorded human works. While it originated as a systematisation of the solutions of practical problems in areas such as land surveying (hence geometry), construction, war and commerce, it evolved with the realisation that abstraction of the essentials led to generalisation of the applications and hence became a science which uses rigorous deduction to arrive at solid conclusions from clearly stated assumptions.

Mathematics is fundamental, not only to much of science and technology, but also to almost all situations that require an analytical model-building approach, whatever the discipline. In recent decades there has been an explosive growth of the use of mathematics in areas outside the traditional base of science, technology and engineering, for example, in finance, biology and computer science.

Statistics as a discipline within mathematics arose from probability and developed in the nineteenth century with the growth of “official statistics”. It now encompasses the whole science of collecting, analysing and interpreting data and the design of observational and experimental studies. It plays a major and increasing role, *inter alia*, in medicine, quality control and management, physical and social sciences, business and economics.

Programmes in mathematics vary from the very pure or theory based to the very applied or practice based. Some are broad, while others allow specialisation in particular areas, such as statistics or financial mathematics. They all share the key learning outcomes detailed below.

Degree profile(s)

Typical degrees offered in the subject area

- First cycle in (name subject area / specific parts)
  - Mathematics
  - Applied Mathematics
  - Mathematical Physics
  - Mathematics and Statistics
  - Mathematics and Education
  - Financial Mathematics/Mathematical Finance
  - Mathematical Engineering

- Second cycle in (name subject area / specific parts)
  - Mathematics
  - Statistics
  - Financial Mathematics
  - Mathematical Engineering
  - Mathematics and Education
  - Biomathematics

- Third cycle in (name subject area / specific parts)
  - Any specialised area of Mathematics

Typical occupations of the graduates in the subject area (map of professions)

First cycle

Programme Profile	Category / Group of professions	Example professions
Mathematics	Industry	Management consultant
Mathematics	Industry	Modeller
Mathematics	Government	Meteorologist
Mathematics with Education	Education	Secondary school teacher of Mathematics
Mathematics specialising in statistics	Industry/Government	Statistician
Mathematics (possibly specialising in finance, statistics or economics)	Banking/Insurance	Actuary, Banker, Accountant
Mathematics with significant computer science	Industry/Banking	Software analyst

Second cycle

Programme profile	Category / Group of professions	Example professions
Mathematics (any specialism)	University	Early Stage Researcher/Teacher
Mathematics	Industry	Management consultant
Mathematics	Industry	Researcher/Modeller
Mathematics	Government	Meteorologist
Mathematics with Education	Education	Secondary school teacher of Mathematics
Mathematics specialising in statistics at second cycle	Industry/Government	Statistician (higher entry level)
Mathematics specialising in finance, statistics or econometrics at second cycle.	Banking/Insurance	Actuary, Banker, Accountant
Mathematics with specialisation at second cycle	Industry/Defence Industry	Researcher

Third cycle

Programme profile	Category / Group of professions	Example professions
Mathematics (any specialism)	University	Researcher/Teacher
Mathematics	Industry	Management consultant
Mathematics	Industry (Pharmaceutical /Aeronautical/Electronic etc)	Researcher/Modeller
Mathematics	Government	Meteorologist
Statistics	Industry, in particular biotechnology and medicine	Researcher/Teacher
Financial or actuarial mathematics	Banking/Insurance	Actuary/Banker
Algebra / Number Theory/Discrete Mathematics	Government	Researcher/cryptologist

Role of subject area in other degree programmes

Mathematics is an essential component of all engineering and most science courses, in particular physics, but also chemistry and, increasingly, biology. Some mathematics units are included in most courses in business studies and economics; statistics has particular importance in these areas and also in programmes in the humanities where no other mathematics courses may form part of the programme.

It is also commonly occurs as one subject in a two subject degree, such as Mathematics and Economics, Mathematics and Computer Science, Mathematics and Biology, Mathematics and Physics.

Learning outcomes & competences - level cycle descriptors

The “Dublin descriptors” describe the generic competences that are expected to be developed in the first, second and third cycles. Mathematics programmes would be expected, of their nature, to develop competences such as “devising and sustaining arguments and solving problems”, while there is a growing awareness of the need to develop communications skills. Similarly, the generic competences identified as most important in the Tuning survey: capacity for analysis and synthesis, capacity to learn and problem solving should all be developed naturally during each of the three cycles. Thus, the generic “Dublin descriptors” and the key Tuning generic competences are assumed at all three cycles, and subject specific descriptors are suggested here for first and second cycles.

The key skills expected of any mathematics graduate are:

- the ability to conceive a proof,
- the ability to model a situation mathematically,
- the ability to solve problems using mathematical tools.

Arising from a survey of academics some subject specific competences were ascribed to each of the first two cycles and are listed here together with suggested cycle and level descriptors.

First cycle descriptors. On successful completion of a first cycle degree in Mathematics, students will be able to:

- show knowledge and understanding of basic concepts, principles, theories and results of mathematics;
- understand and explain the meaning of complex statements using mathematical notation and language;
- demonstrate skill in mathematical reasoning, manipulation and calculation;
- construct rigorous proofs;
- demonstrate proficiency in different methods of mathematical proof.

First cycle level descriptor.

Level 1. *Content.* The Mathematics all scientists should know: basic algebra and arithmetic, linear algebra, calculus, basic differential equations, basic statistics and probability.

Learning outcomes. To complete level 1, students will be able to
(a) understand some theorems of Mathematics and their proofs;
(b) solve mathematical problems that, while not trivial, are similar to others previously known to the students;
(c) translate into mathematical terms simple problems stated in non-mathematical language, and take advantage of this translation to solve them.

Level 2. *Content.* Basic theory of the main "mathematical subjects" including most, preferably all, of the following:

- basic differential equations
- basic complex functions
- some probability
- some statistics
- some numerical methods
- basic geometry of curves and surfaces
- some algebraic structures
- some discrete mathematics.

Learning outcomes. To complete level 2, students will be able to
(d) provide proofs of mathematical results not identical to those known before but clearly related to them;
(e) translate into mathematical terms problems of moderate difficulty stated in non-mathematical language, and take advantage of this translation to solve them;
(f) solve problems in a variety of mathematical fields that require some originality;
(g) build mathematical models to describe and explain non-mathematical processes.

Some first cycle subject specific competences:

- Profound knowledge of “elementary” mathematics (such as may be covered in secondary education).
- Ability to construct and develop logical mathematical arguments with clear identification of assumptions and conclusions.
- Capacity for quantitative thinking.
- Ability to extract qualitative information from quantitative data.
- Ability to formulate problems mathematically and in symbolic form so as to

facilitate their analysis and solution.

- Ability to design experimental and observational studies and analyse data resulting from them.
- Ability to use computational tools as an aid to mathematical processes and for acquiring further information.
- Knowledge of specific programming languages or software.

Second cycle descriptor.

Learning outcomes. On successful completion of a second cycle degree in Mathematics, students will be able to:

- read and master a topic in the mathematical literature and demonstrate mastery in a reasoned written and/or verbal report;
- initiate research in a specialised field.

<u>Some second cycle subject specific competences:</u>

- Facility with abstraction including the logical development of formal theories and the relationships between them.
- Ability to model mathematically a situation from the real world and to transfer mathematical expertise to non-mathematical contexts.
- Readiness to address new problems from new areas.
- Ability to comprehend problems and abstract their essentials.
- Ability to formulate complex problems of optimisation and decision making and to interpret the solutions in the original contexts of the problems.
- Ability to present mathematical arguments and the conclusions from them with clarity and accuracy and in forms that are suitable for the audiences being addressed, both orally and in writing.
- Knowledge of the teaching and learning processes of mathematics.

<u>Consultation process with stakeholders</u>

Graduates and employers were consulted about generic, and some subject specific competences in a questionnaire as part of Tuning I; academics were consulted also about more detailed subject specific competences. The results informed the subsequent paper “Towards a common framework for Mathematics degrees in Europe”, which was also published in the Bulletin of the European Mathematical Society. This paper has also been widely disseminated at national levels.

Workload and ECTS

While most of the European higher education area is tending towards 180 ECTS credit first cycle degrees, Spain, Portugal and Ireland, for various reasons, not least the age of entry, will most likely have a preponderance of 240 ECTS credit first cycle programmes. The Tuning group expressed the view that if a paedagogical qualification were to be obtained as part of the first cycle, then it should have 240 ECTS credits. The requirement that a thesis or dissertation form a significant part of the second cycle suggests a range of 90 to 120 credits at this cycle.

Learning, teaching & assessment

Learning and teaching of mathematics typically involves a combination of the following:

- **Lectures.** These are seen as a very time-efficient way for students to learn part of the large material involved in the corpus of mathematics. In some cases, students acquire prepared lecture notes or have a set textbook; in other cases the taking of notes is seen as part of the learning process.

- **Exercise sessions**. These are organised most often in tandem with lectures. They occur as groups with supervision, or individually as homework with subsequent supervision of the results. The aim of the exercises is two-fold: understanding of the theoretical material through examples and applications to problems. These sessions are essential in mathematics, where understanding is acquired by practice, not memorisation.

- **Homework.** While demanding on the time of the lecturer and/or teaching assistant, homework is clearly one of the most effective ways in which students can be encouraged to explore the limits of their capabilities. Homework, of course, allows feedback to the students, which gives them a clearer picture of their performance; however, while homework is often assigned, it is less often graded, except where classes are small.

- **Computer laboratories.** These are perhaps the most significant change in the teaching of mathematics in recent years, introducing an experimental aspect to the subject. They feature not only in computer science related and computational courses, but also in statistics, financial mathematics, dynamical systems etc.

- **Projects**. These are done individually or in small groups, and typically involve putting together material from different sub-fields to solve more complicated problems. Small group projects can help to develop the ability to do teamwork (identified as an important transferable skill). The projects may involve significant computational elements, as in the case of the computational competences referred to above. Projects, particularly significant final year projects where they exist, also afford the opportunity to develop students' verbal and written communications skills.

- **Dissertation.** This is considered essential at second cycle, where it should be supplemented with bibliographic searches etc.

Examples of best practice in learning, teaching and assessment for some subject specific competences:

Competence: Ability to formulate problems mathematically and in symbolic form so as to facilitate their analysis and solution.

This competence essentially involves the ability to express a simple problem in the form of an equation, to express a statement written in common language in symbolic/mathematical form and vice versa, and to be critical about the solution: to know when a solution is sensible. This can be developed through feedback on exercises, and through problem solving and project work, where the application can illustrate the reasonableness of the solution.

Competence: Ability to design experimental and observational studies and analyse data resulting from them.

One interpretation of this competence is that first cycle students should be able to design functioning code segments in a high level language, correct input errors (i.e., understand the mathematics of the syntax), and then interpret the data (for example a phase plane portrait). In general, as computer aided analysis becomes more and more common, ability to appropriately design experiments will become a skill of increasing importance. Laboratory sessions are the best environment in which to develop such skills.

Competence: Facility with abstraction including the logical development of formal theories and the relationships between them.

This includes the following “abilities”:

- understanding what mathematical objects are,
- manipulating them under formal rules,
- distinguishing between correct and incorrect operations,
- understanding the role of axioms, definitions and theorems.

Students are introduced to a variety of formal mathematical theories. They explore the limits of the theories under study, and they learn how some aspects of reality can be transformed into a formal theory after excluding what is considered accidental for the particular problem. They study and understand some theorems, perform some manipulations under formal rules and check their work against the correct versions, which are supplied.

Quality enhancement

The “Tuning checklist for programme development and quality enhancement in the framework of the Bologna reform” provides a mechanism for the development of new programmes. It also serves as a reference point for modifications of existing programmes: e.g., redrafting in terms of learning outcomes and ECTS credits.