

SUMMER SCHOOL IN COMPLEX GEOMETRY AT MILANO BICOCCA

June 23rd-28th, 2014

UNDER THE AUSPICES AND WITH THE SUPPORT OF THE PhD PROGRAM IN PURE AND APPLIED MATHEMATICS OF THE UNIVERSITA' DI MILANO BICOCCA

'ASYMPTOTIC ASPECTS OF COMPLEX AND ALGEBRAIC GEOMETRY'

Lecturers: **JEAN-PIERRE DEMAILLY** (Grenoble), **ROBERT LAZARSFELD** (Stony Brook), **STEVE ZELDITCH** (Northwestern)

The summer-school "*Asymptotic aspects of complex and algebraic geometry*" will take place at the Department of Mathematics and Applications, Università di Milano Bicocca, Milano, Italy during the period June 23rd-28th, 2014. This school is organized under the auspices of the PhD program in Mathematics at the Università di Milano Bicocca.

Goal of the school: The broad focus of the school is on the asymptotic manifestations of positivity in Kähler and algebraic geometry, viewed from different algebraic and analytic perspectives. The school is specifically targeted at advanced undergraduate students, PhD students and young researchers that have a general interest in either complex differential or algebraic geometry, but should be of great interest to more experienced researchers as well.

In the last two decades, there has been a vast amount of research on asymptotic aspects in complex and algebraic geometry variously related to positivity concepts, ranging from, say, Bergman-Szegő kernels in Kähler geometry to Boij-Södenbergh theory of syzygies in algebra. All these ideas have brought into light many asymptotic tools, and uncovered new insights of analytic nature like approximation of metrics, of geometric nature like positivity aspects and vanishing theorems, and of algebraic nature like the study of syzygies of an algebraic variety. The following three introductory courses by leading researchers will offer an introduction to this fast growing and very fertile area of research.

Description of the courses:

(1) **Jean-Pierre Demailly** (Université de Grenoble, France)

Title: *Kähler geometry, positivity aspects, structure theorems.*

Abstract:

The goal of the talks will be to present a few basic techniques in complex analytic geometry, and their applications to the study of projective or compact Kähler manifolds. Especially, we will present various results concerning positive cohomology classes, some recent vanishing theorems assuming only semipositivity hypotheses, and related structure theorems for compact Kähler manifolds satisfying suitable curvature conditions, e.g. semipositivity of Ricci-curvature. Asymptotic approximation techniques for plurisubharmonic functions and the related intersection theory of closed positive currents play an important role in the proofs.

(2) **Robert Lazarsfeld** (Stony Brook University, USA)

Title: *Asymptotic properties of the defining equations and higher syzygies of projective varieties.*

Abstract:

The equations defining projective varieties have been of interest to algebraic geometers and algebraists both classically and in more recent times. In the early 1980s, Green and others realized that classical results concerning defining equations should be seen as the first cases of more general statements about higher syzygies. This has sparked a great deal of work over the last twenty-five years, bring to bear many geometric techniques including vanishing theorems, vector bundles, and Hilbert schemes, among other tools. Very recently, there have been two new developments concerning the asymptotic properties of syzygies as the positivity of the embedding increases. First, some old questions involving the one-dimensional case have been answered. Secondly, the picture in higher dimensions has started to come into focus. But there remain many open questions. I will survey this circle of ideas.

(3) **Steve Zelditch** (Northwestern University, USA):

Title: *Bergman kernels, Bergman metrics and their applications.*

Abstract:

This mini-course is about holomorphic sections of high powers of a positive line bundle $(L, h) \rightarrow M$ over a Kähler manifold. The key analytic objects are the Bergman kernels B_k , i.e. the orthogonal projection onto the holomorphic sections of the k -th power of L with respect to an inner product induced by the Hermitian metric h . Bergman kernels induce Bergman metrics of degree k , which are special algebro-geometric metrics induced by holomorphic embeddings into projective space. Over the last 15 years, Bergman metrics have been used to approximate general metrics in an even more precise way and the space of Bergman metrics (a symmetric space of type $SL(N, \mathbf{C})/SU(N)$) gives a geometric approximation to the infinite dimensional symmetric space of all Kähler metrics. Geodesics in the space of Kähler metrics are solutions of a homogeneous complex Monge Ampere equation. The purpose of these lectures is to start with the construction of the Bergman kernel and to explain how the geometry of the space of Bergman metrics (e.g. its geodesics) approximates the geometry of the infinite dimensional space.

More info: Updated information regarding schedule and location as well as social activities will be made available from the web page of the Dipartimento di Matematica e Applicazioni,

<http://home.matapp.unimib.it/>

The website of the summer school will be soon available.

No registration fee is required; nonetheless, an informal registration is welcome. This may help the organizers to run smoothly the school and its related social activities. The interested participants should communicate their intention to attend, by simply sending an e-mail to one of the the organizers.

In addition to the courses a limited number of seminars on related topics may be offered. Participants interested in giving a seminar should inform the organizers.

Regrettably, no financial assistance for participants is foreseen at the moment.

Scientific Committee:

The conference is supervised by the complex/symplectic geometry group of the PhD program in Mathematics at Milano Bicocca:

Roberto Paoletti (roberto.paoletti@unimib.it) at Milano Bicocca, Costantino Medori (costantino.medori@unipr.it) and Adriano Tomassini (adriano.tomassini@unipr.it) at Parma

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