The Infinity Computer and numerical computations with infinities and infinitesimals

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The lecture introduces a new methodology allowing one to execute numerical computations with finite, infinite, and infinitesimal numbers (see [1–19]) on a new type of a computer – the Infinity Computer (see EU, USA, and Russian patents [8]). The new approach is based on the principle 'The whole is greater than the part' (Euclid's Common Notion 5) that is applied to all numbers (finite, infinite, and infinitesimal) and to all sets and processes (finite and infinite). It is shown that it becomes possible to write down finite, infinite, and infinitesimal numbers by a finite number of symbols as particular cases of a unique framework different from that of the non-standard analysis. The new computational methodology (see survey [9]) evolves ideas of Cantor and Levi-Civita in a more applied way and, among other things, introduces new infinite integers that possess both cardinal and ordinal properties as usual finite numbers (its relations with traditional approaches are discussed in [4,5]).

It is emphasized that the philosophical triad – researcher, object of investigation, and tools used to observe the object – existing in such natural sciences as Physics and Chemistry, exists in Mathematics, too. In natural sciences, the instrument used to observe the object influences the results of observations. The same happens in Mathematics where numeral systems used to express numbers are among the instruments of observations used by mathematicians. The usage of powerful numeral systems gives the possibility to obtain more precise results in Mathematics, in the same way as the usage of a good microscope gives the possibility to obtain more precise results in Physics. A new numeral system allowing one to express easily infinities and infinitesimals offers exciting capabilities for describing mathematical objects, mathematical modeling, and practical computations. The concept of the accuracy of numeral systems is introduced. The accuracy of the new numeral system is compared with traditional numeral systems used to work with infinity.

The new computational methodology gives the possibility to execute computations of a new type and simplifies fields of Mathematics where the usage of the infinity and/or infinitesimals is necessary (e.g., divergent series, limits, derivatives, integrals, measure theory, probability theory, optimization, fractals, etc., see [1–7,9–19]). Numerous examples and applications are given. In particular, a number of results related to the First Hilbert Problem and Turing machines are established.

The main attention in the lecture is dedicated to the explanation of how practical numerical computations with infinities and infinitesimals can be executed (e.g., see [2,13,16] for applications in optimization, numerical differentiation, and ODEs). The Infinity Calculator using the Infinity Computer technology is presented during the talk.

Numerous research articles of several authors and a lot of an additional information can be downloaded from the page http://www.theinfinitycomputer.com

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