ON THE BOUNDED HEIGHT CONJECTURE

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Date: **June 22 and 24, 2009** Time: **tba** Place: **tba**

In 1999 Bombieri, Masser, and Zannier studied the intersection of an irreducible algebraic curve $\mathcal{C}$ contained in the algebraic torus with the union of all algebraic subgroup of codimension at least r. They obtained two results. First, assume $r = 1$ and that $\mathcal{C}$ is not contained in the translate of a proper algebraic subgroup. Then there is a constant $B = B(\mathcal{C})$ such that any point on $\mathcal{C}$ contained in the union of all algebraic subgroups of codimension at least 1 (i.e. all proper algebraic subgroups) has absolute height bounded by B. In general the set thus obtain is infinite. Second, if we assume $r = 2$, hence we restrict to taking the union over all codimension 2 algebraic subgroups, then we are left with a finite set.

In subsequent work these authors conjectured an analogy of the statement given above on bounded height but with $\mathcal{C}$ replaced by a subvariety $\mathcal{X}$ of any dimension. In simplified terms they expected that there is a natural and closed subvariety $\mathcal{Z} \subseteq \mathcal{X}$ and a constant $B = B(\mathcal{X})$ such that any point in $\mathcal{X} \backslash \mathcal{Z}$ contained in the union of all algebraic subgroups of codimension at least $\dim \mathcal{X}$ has absolute height uniformly bounded by B.

In these two 90 minute lectures I will give an overview of the history of this problem and an outline of a proof of the conjecture for varieties of arbitrary dimension. The method involves an algebraic and an analytic part. In the algebraic part we will be concerned with finding a suitable compactification of $\mathcal{X}$ with respect to a fixed algebraic subgroup. Having done this the powerful machinery of height functions and intersection theory for projective varieties will lie at our disposal. The analytic part studies the interplay of analytic subgroups and our variety $\mathcal{X}$. In a certain sense the set of analytic subgroups is just a compactification of the set of algebraic subgroup. An important tool here is a theorem of Ax which enables us to translate information coming from analytic subgroups into algebraic terms.

The main technical tools such as heights and analytic subgroups will be introduced.