Canonical Heights in Arithmetic Dynamical Systems

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Iteration of a morphism $\phi : \mathbb{P}^N \to \mathbb{P}^N$ leads to a discrete dynamical system. Classically such dynamical systems have been studied for $\mathbb{P}^N(\mathbb{C})$, but if we instead take a morphism defined over a number field K, then there are many interesting arithmetic questions associated to iteration of the map $\phi : \mathbb{P}^N(\bar{K}) \to \mathbb{P}^N(\bar{K})$. In particular, Tate's construction gives a canonical height function $\hat{h}_\phi(P) = \lim_{n\to\infty} d^{-n} h\big(\phi^{\circ n}(P)\big)$ on $\mathbb{P}^N(\bar{K})$ whose values are intimately connected to the arithmetic properties of the dynamical system attached to ϕ. In this talk I will survey the theory of these dynamical canonical heights and then discuss recent joint work with Shu Kawaguchi on the question of when two morphisms can have identical canonical heights.