A quantitative version of the Soap Bubble Theorem

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Abstract

The celebrated *Soap Bubble Theorem* of Alexandrov asserts that round spheres are the only closed constant mean curvature hypersurfaces embedded in the Euclidean space. The talk mainly focuses on the following quantitive version of the theorem:

Theorem [Ciraolo - V.]**.** *Let S be an n-dimensional, C^2-regular, connected, closed hypersurface embedded in the Euclidean space. There exist constants ϵ, $C > 0$ such that if*

$$\mathrm{osc}(H) \leq \epsilon,$$

then there are two concentric balls B_{r_i} and B_{r_e} such that

$$S \subset \overline{B}_{r_e} \setminus B_{r_i},$$

and

$$r_e - r_i \leq C \mathrm{osc}(H).$$

The constants ϵ and C depend only on n and upper bounds on the C^2-regularity and the area of S.

The proof of the theorem makes use of a quantitive study of the method of the moving planes and the result implies a new pinching theorem for hypersurfaces in the Euclidean space. Furthermore, the theorem is optimal in a sense that it will be specified in the talk.

The last part of the talk will be about an on-going study on the generalization of the result in space forms or, more generally, in warped product spaces.