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Title: Efficiency and localisation for the first Dirichlet eigenfunction
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Abstract Bounds are obtained for the efficiency or mean to peak ratio $E(\Omega)$ for the first Dirichlet eigenfunction for open, connected sets Ω with finite measure in Euclidean space \mathbb{R}^m . It is shown that (i) localisation implies vanishing efficiency, (ii) a vanishing upper bound for the efficiency implies localisation, (iii) localisation occurs for a wide class of elongating bounded, open, convex and planar sets, (iv) the efficiency of any quadrilateral with perpendicular diagonals of lengths 1, and n respectively is $O(n^{-2/3} \log n)$ as $n \rightarrow \infty$, and (v) the efficiency of $\{(x_1, x_2) : (2|n^{-1}x_1|)^\alpha + (2|x_2|)^\alpha < 1\}$, $1 \leq \alpha < \infty$, is $O(n^{-2/(\alpha+2)}(\log n)^{\max\{1/\alpha, 1/2\}})$, $n \rightarrow \infty$. This disproves some claims in the literature. A key technical tool is the Feynman-Kac formula. Joint work with F. Della Pietra, G. di Blasio, N. Gavitone.