On the existence and regularity of solutions of the Born-Infeld equation

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Abstract

In this talk, we investigate solutions of the electrostatic Born-Infeld model

$$\begin{cases} \operatorname{div}\left(\dfrac{Du}{\sqrt{1-|Du|^2}}\right) = -\rho \qquad \text{on } \Omega \subset \mathbb{R}^N \\ u = \phi \qquad \text{on } \partial\Omega, \end{cases} \tag{$\mathcal{BI}$}$$

where ρ is a distribution and $\phi \in \mathrm{Lip}(\partial\Omega)$ is spacelike (for instance, it has Lipschitz constant strictly less than 1). The above can be read as a prescribed Lorentzian mean curvature equation for the graph of u in Minkovski space $\mathbb{L}^{N+1}$. Formally, $(\mathcal{BI})$ is the Euler-Lagrange equation for the functional

$$\mathscr{F}(u) = \int_\Omega \left(1 - \sqrt{1-|Du|^2}\right) - \langle \varrho, u \rangle$$

in the appropriate function space of 1-Lipschitz functions. While $\mathscr{F}$ is easily seen to admit a unique minimizer u_ρ, it is still an open problem whether u_ρ solves $(\mathcal{BI})$ for any chosen ρ, the main issue being the presence of light segments in the graph of u_ρ, that is, segments in Ω where u is linear with slope 1.

The question has been addressed only in recent years, and a positive answer is known in very few special cases. In this talk, we assume

$$\rho = \sum_{i=1}^{k} a_i \delta_{x_i} + H\mathrm{d}x \qquad \text{with } a_i \in \mathbb{R}, \ \int_\Omega H^2 < \infty,$$

and show that u solves $(\mathcal{BI})$, has no light segments, and that the graph of u has second fundamental form in L^2_{loc} on $\Omega\backslash\{x_1, \ldots, x_k\}$. In particular, u satisfies the enhanced $W^{2,2}_{\mathrm{loc}}$ estimates

$$\int_{\Omega'} \frac{|D^2u|^2}{\sqrt{1-|Du|^2}}\mathrm{d}x \le C_{\Omega'} \qquad \text{on } \Omega' \Subset \Omega\backslash\{x_1, \ldots, x_k\}.$$

This is joint work with J.Byeon, N. Ikoma and A. Malchiodi.