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Report on Raffaele Marcovecchio's application for the Ministry of Education

To whom it may concern.

I understand that the Italian government has recently decided to create some special positions in the universities, for a period up to six years, reserved for young Italian scientists working abroad, with the aim of encouraging them to return and make their research in Italy. Since Raffaele Marcovecchio would be entitled for such a position, I am pleased to write this letter and to support his application.

His work is devoted to that part of number theory which is called Diophantine Approximation. The main goal is to study the arithmetic nature of constants arising from analysis. To prove the irrationality of such constants is most often a very challenging task. There are plenty of difficult open problems in this domain. For those constants for which irrationality has been proved, a fundamental question is to give an irrationality measure. For x a real irrational number, one wishes to find an exponent μ such that

$$\left|x - \frac{p}{q}\right| > \frac{1}{q^\mu}$$

for all rational number p/q with sufficiently large denominator q. The smallest μ, the sharper the estimate. Such quantitative statements have a large range of applications, but researcher in number theory study these questions for their own sake; they represent a challenge which is also a test for new ideas. Very sophisticated tools have been introduced, refinements require intricate proofs, combining ingenious ideas with technical skills. The least improvement requires new ideas. Often, the estimate remains as a record during long periods. To break such a record is very hard.

In 2008, Raffaele Marcovecchio succeeded in proving the above irrationality estimate for the number $x = \log 2$ with the exponent $\mu = 3.57$. The previous record was due to E.A. Rukhadze in 1987 who had only the exponent 3.89. This remarkable achievement by Marcovecchio involves a number of tools, he combines ideas introduced previously by other authors for this problem as well as for similar ones, with new ideas of his own. Among the tools he uses is a fundamental one due to Rhin and Viola involving the action of a group; they introduced these actions for the study of zeta values. Before Marcovecchio, nobody succeeded to use such a tool for improving the irrationality measure of $\log 2$.

This improvement of the irrationality measure of $\log 2$ is by far the most impressive achievement of Marcovecchio: many specialists would be happy to have such a record in their cv. His result is known worldwide, as I could witness during recent visits I had to a number of different countries. However, this is certainly not the only contribution of Marcovecchio to the subject, and his other works deserve to be quoted: they provide evidence that he is a promising young mathematician who is likely to produce further valuable new results.

Apart from his fundamental Acta Arithmetica paper which appeared in 2009, he already has two other publications in refereed journals, which appeared in 2004 and 2006. In the first one, dealing with exponential polynomials, he produces an upper bound for the number of solutions of an exponential Diophantine equation. The proof is an extension of a method due to Schlickewei and Viola, the main tool is Schmidt's Subspace Theorem. In the second paper, he proves that for α an algebraic number in the domain $0 < |\alpha| < 1$, the dimension of the $\mathbf{Q}(\alpha)$–vector space spanned by the numbers 1, $\mathrm{Li}_1(\alpha)$, $\mathrm{Li}_2(\alpha), \ldots$ has infinite dimension. Here, Li_s denotes the function

$$\mathrm{Li}_s(z) = \sum_{n\geq 1} \frac{z^n}{n^s}\cdot$$

This is already a remarkable result, it is published in one of the best journals (Ann. Scuola Norm. Sup. Pisa).

His thesis (University of Pisa, November 2004, under the joint supervision of Francesco Amoroso and Carlo Viola), also includes an unpublished work on the linear independence of 1, $\mathrm{Li}_1(\alpha)$, $\mathrm{Li}_2(\alpha)$ for various special algebraic numbers α.

His research program is at the same time ambitious and realistic. It is amazing that the methods which have been derived from Apéry's proof of the irrationality of $\zeta(3)$ do not work for $\zeta(4)$. He is working in this direction with Christian Kattenthaler and Wadim Zudilin. His program also includes a project of joint work with Carlo Viola. As a matter of fact, he has no joint paper so far. He is now going to prove his ability to work with other colleagues.

In conclusion, Raffaele Marcovecchio is a very strong candidate for the above mentioned program, and I support his application.

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