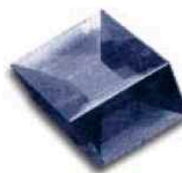


Matematica

Dipartimento

Università degli Studi di Roma Tor Vergata



**AVVISO DI SEMINARIO  
ELENA VILLA**

Università degli Studi di Milano

21 aprile 2016  
ore 15 Aula D'Antoni

**ASYMPTOTIC RESULTS FOR ESTIMATORS OF THE MEAN DENSITY OF RANDOM  
CLOSED SETS AND OF THE INTENSITY OF POINT PROCESSES**

Many real phenomena may be modelled as random closed sets in  $\mathbb{R}^d$ , of different Hausdorff dimensions. Of particular interest are cases in which their Hausdorff dimension, say  $n$ , is strictly less than  $d$ , such as fiber processes, boundaries of germ-grain models, and  $n$ -facets of random tessellations. The mean density, say  $\lambda_{\Theta_n}$ , of a random closed set  $\Theta_n$  in  $\mathbb{R}^d$  with Hausdorff dimension  $n$  is defined as the density of the measure  $\mathbb{E}[\mathcal{H}^n(\Theta_n \cap \cdot)]$  with respect to  $\mathcal{H}^d$ , whenever it exists. A crucial problem is the pointwise estimation of  $\lambda_{\Theta_n}$ .

In this talk we present two different kinds of estimators of  $\lambda_{\Theta_n}(x)$ . The first one follows as a generalization to the  $n$ -dimensional case of the classical kernel density estimator of random vectors, and so, even if it is not of easy applicability when  $n > 0$ , it finds its interest under a more theoretical point of view. The second one follows by a local approximation of  $\lambda_{\Theta_n}$  based on a stochastic version of the  $n$ -dimensional Minkowski content of  $\Theta_n$  and reveals its benefits in applications in the non-stationary case. We introduce the notion of the optimal bandwidth, deriving explicit expressions for both of them. We analyze the asymptotical properties of the two above mentioned estimators; in particular, by means of large and moderate deviation results, we show that the "Minkowski content"-based estimator for the mean density of a random closed set is strongly consistent and asymptotically Normal if the optimal bandwidth is employed. Besides confidence regions for the mean density of the involved random closed set in  $m \geq 1$  distinct points  $x_1, \dots, x_m$  are provided. Finally, we try to develop a similar approach to the kernel estimator as well, by facing the estimation of the intensity function of a point process, being a point process a particular random closed set with dimension  $n = 0$ . More specifically in such a situation the kernel estimator turns out to be strongly consistent and asymptotically Normal, generalizing the results known in literature for random vectors.

Ciclo di Seminari - Progetto ERC grant 277742 PASCAL



European  
Research  
Council

Via della Ricerca Scientifica, 1 - 00133 Roma