Matematica
Dipartimento
Università degli Studi di Roma Tor Vergata

AVVISO DI SEMINARIO
ELENA VILLA

Università degli Studi di Milano

21 aprile 2016
ore 15 Aula D'Antoni

ASYMPTOTIC RESULTS FOR ESTIMATORS OF THE MEAN DENSITY OF RANDOM CLOSED SETS AND OF THE INTENSITY OF POINT PROCESSES

Many real phenomena may be modelled as random closed sets in $\mathbb{R}^d$, of different Hausdorff dimensions. Of particular interest are cases in which their Hausdorff dimension, say n, is strictly less than d, such as fiber processes, boundaries of germ-grain models, and n-facets of random tessellations. The mean density, say λ_{Θ_n}, of a random closed set Θ_n in $\mathbb{R}^d$ with Hasudorff dimension n is defined as the density of the measure $\mathbb{E}[\mathcal{H}^n(\Theta_n \cap \cdot)]$ with respect to $\mathcal{H}^d$, whenever it exists. A crucial problem is the pointwise estimation of λ_{Θ_n}.

In this talk we present two different kinds of estimators of $\lambda_{\Theta_n}(x)$. The first one follows as a generalization to the n-dimensional case of the classical kernel density estimator of random vectors, and so, even if it is not of easy applicability when $n > 0$, it finds its interest under a more theoretical point of view. The second one follows by a local approximation of λ_{Θ_n} based on a stochastic version of the n-dimensional Minkowski content of Θ_n and reveals its benefits in applications in the non-stationary case. We introduce the notion of the optimal bandwidth, deriving explicit expressions for both of them. We analyze the asymptotical properties of the two above mentioned estimators; in particular, by means of large and moderate deviation results, we show that the "Minkowski content"-based estimator for the mean density of a random closed set is strongly consistent and asymptotically Normal if the optimal bandwidth is employed. Besides confidence regions for the mean density of the involved random closed set in $m \geq 1$ distinct points $x_1, \dots x_m$ are provided. Finally, we try to develop a similar approach to the kernel estimator as well, by facing the estimation of the intensity function of a point process, being a point process a particular random closed set with dimension $n = 0$. More specifically in such a situation the kernel estimator turns out to be strongly consistent and asymptotically Normal, generalizing the results known in literature for random vectors.

Ciclo di Seminari - Progetto ERC grant 277742 PASCAL

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