International Workshop

New challenges in reciprocal processes, Schrödinger bridges and optimal transport with applications to control engineering problems for classical and quantum systems

Organizers: Paolo Dai Pra, Michele Pavon and Francesco Ticozzi [1]

The workshop will take place at the Department of Mathematics, University of Padova, via Trieste 63, 35131 Padova, on *Friday May 29th, 2015* in rooms 1BC45 (morning) and 1BC50 (afternoon).

Tentative Program

Yongxin Chen
Department of Electrical and Computer Engineering
University of Minnesota

Schrödinger bridges and the steering of stochastic and deterministic systems

Abstract: We present an overview of our recent work on implementable solutions to the Schrödinger bridge problem and their potential application to stochastic optimal control, optimal transport, and various generalizations. In particular, we discuss the case of degenerate constant diffusion coefficients and the steering of linear dynamical systems between two one-time state-distributions using state feedback, the limiting case of Optimal Mass transport with nontrivial prior dynamics. For the special case of Gaussian marginals, closed-form solutions will be presented.
The presentation is based on joint work with Tryphon T. Georgiou and Michele Pavon.

Giovanni Conforti
Institut für Mathematik
Universität Potsdam

Reciprocal characteristics, lattices, and concentration of measure

Abstract: In this talk we are interested in establishing concentration inequalities for a Poisson random vector conditioned to belong to some linear subspace. The study of the reciprocal class of a random walk, offers, as a by product, some interesting characterizations of such distributions. We will first revisit the concentration properties that can be deduced from the Modified Logarithmic Inequality, showing that such inequality actually gives sharp bound for the Poisson distribution. Motivated by some remarks which link conditioning and curvature for absolutely continuous random vectors, we derive concentration bounds for pinned discrete random vectors. As it often happens, the discrete case is trickier. However, some nice connections with lattice geometry allow to overcome the main difficulties. Finally we show that the bounds we obtain are sharp in some examples, and that methods we used are robust, i.e. they may be used outside the Poissonian framework.

[1]P. Dai Pra and M. Pavon are with the Department of Mathematics, University of Padova, F. Ticozzi is with the Department of Information Engineering, University of Padova

Markus Fischer
Department of Mathematics
University of Padova

On large deviations for the empirical measures of weakly interacting systems

Abstract: One of the basic results of large deviations theory is Sanov's theorem, which states that the sequence of empirical measures of independent and identically distributed samples satisfies the large deviation principle with rate function given by relative entropy with respect to the common sample distribution. The large deviation principle for the empirical measures is known to hold also for broad classes of weakly interacting systems (or mean field systems). For systems where the interaction through the empirical measure corresponds to an absolutely continuous change of measure, one can express the large deviations rate function as relative entropy of a distribution with respect to the law of the McKean-Vlasov limit with measure-variable frozen at that distribution. This form of the rate function is a natural generalization of Sanovs theorem. We discuss situations (beyond that of tilted distributions) in which the large deviation principle holds with rate function in relative entropy form.

Tryphon T. Georgiou
Department of Electrical and Computer Engineering
University of Minnesota

The Hilbert metric and Schrödinger bridges

Abstract: Hilberts projective metric was introduced in 1895. In the 1950s Garrett Birkhoff and Hans Samelson realized the significance of the metric in proving the existence of positive eigenvectors to linear operators that leave a cone invariant via a contraction principle; Birkhoffs version of the metric was further developed for nonlinear operators by Bushell. The purpose of this talk is to draw attention to this circle of ideas which, in the same spirit, help ascertain but also construct solutions to Schroedinger-type systems of equations that arise in the bridge problem. Thus, we first address a basic version of Schroedinger's bridge problem for Markov chain. Then, building on the work Pavon and Ticcozzi, we discuss a non-commutative version to determine a suitable update of a quantum channel (Kraus map) so as to make it consistent with given marginal density matrices. Finally, we explain how the contractive principle in the Hilbert geometry may be employed to construct solutions to Schroedinger's problem more generally, as well as for the limiting case of Optimal Mass Transport.
The presentation is based on joint work with Yongxin Chen and Michele Pavon.

Christian Leonard
Département de Mathématiques et Informatique
Université Paris Ouest

Looking at the Schrödinger problem with calculus of variations in mind

Abstract: Minimizers of the dynamical Schrdinger problem are variational path measures. They are stochastic analogues of the deterministic solutions of the action minimizing Hamilton principle and of the Monge-Kantorovich optimal transport problem. With this analogy in mind, the analogue of gauge invariance will be presented. We shall also compute the (Krener) characteristics of the reciprocal class of diffusion path measures and random walks on graphs.
This lecture is based on joint works with Giovanni Conforti and Jean-Claude Zambrini

Sylvie Roelly
Institut für Mathematik
Universität Potsdam

A stochastic analysis approach for the characterization of reciprocal classes: the case of Brownian diffusions and of compound Poisson processes.

Abstract: Reciprocal processes (whose concept can be traced back to E. Schrödinger) form a class of stochastic processes constructed as mixture of bridges, that satisfy a temporal Markovian field property. We discuss an unified approach to characterize different types of reciprocal processes via duality formulae on path spaces. Two main examples are treated: the case of reciprocal processes with continuous paths associated to Brownian diffusions and the case of reciprocal processes associated to pure jump processes.
This talk is based on joint works with M. Thieullen, R. Murr, C. Lonard, P. Dai Pra and G. Conforti.

Francesco Ticozzi
Department of Information Engineering
University of Padova

A walk to symmetrization via a Schroedingers bridge

Abstract: Symmetrizing dynamics are ubiquitous in physics, communication and control engineering: for example, various classes of thermalization, noise rejection, coordination or randomization tasks can be reformulated as symmetrization problems with respect to suitable groups of transformation. Recently, a unifying approach to address such problems with robust and randomized algorithms has been proposed, building on the ideas underlying the so-called gossip dynamics in networked systems. In this talk we show how a Schroedingers bridge could be used to design dynamics that achieve the sought-after symmetrization in finite time, while respecting the constraints of the problem at hand, and discuss the key issues in determining their performance in particular those related to optimality and computational complexity.

Lorenza Viola
Department of Physics and Astronomy
Dartmouth College

To be confirmed