**Weak Dirichlet processes, path-dependent martingale problems and BSDEs.**

*Part I. The continuous framework and BSDEs with distribution driver.*
*Part II. Discontinuous weak Dirichlet processes and path-dependent martingale problem.*

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The notion of weak Dirichlet process is the natural extension of the concept of semimartingale. Among the examples we find the following.

1. Irregular Markov processes solutions of SDEs with distributional drift with jumps, BSDEs with distributional driver.
2. Solutions of (even discontinuous) path-dependent SDEs with distributional drift.
3. (Path-dependent) Bessel processes. Identification problem in BSDEs driven by random measure.

The first part of the talk concentrates on motivations and fundamental concepts: in particular we provide a new decomposition for weak Dirichlet processes, which is also new for semimartingales. Later we concentrate on BSDEs with distributional driver.

In the second part we investigate a set of new useful chain rules. In particular we introduce the concept of characteristics for a weak Dirichlet process and discuss a general framework of (possibly path-dependent with jumps) martingale problem.