Distribution of the Height of Local Maxima of Gaussian Random Fields

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Let $\{f(t) : t \in T\}$ be a smooth Gaussian random field over a parameter space T, where T may be a subset of Euclidean space or, more generally, a Riemannian manifold. We provide a general formula for the distribution of the height of a local maximum

$$\mathbb{P}\{f(t_0) > u | t_0 \text{ is a local maximum of } f(t)\}$$

when f is non-stationary. Moreover, we establish asymptotic approximations for the overshoot distribution of a local maximum

$$\mathbb{P}\{f(t_0) > u + v | t_0 \text{ is a local maximum of } f(t) \text{ and } f(t_0) > v\}$$

as $v \to \infty$. Assuming further that f is isotropic, we apply techniques from random matrices theory related to the Gaussian orthogonal ensemble to compute such conditional probabilities explicitly when T is Euclidean or a sphere of arbitrary dimension. Such calculations are motivated by the statistical problem of detecting peaks in the presence of smooth Gaussian noise.