Roma May and June 2023

Amine Asselah *

We propose a graduate course of about 16 hours on transient random walks, simple or branching, runing over a period of four weeks.

Simple Random Walk is the first example of a Markovian process, whereas Branching Random Walk has a time indexed by a random tree, and looses the SRW's sequential way of visiting space.

The highlights of the course would be two phenomena in high dimensions linked with folding.

- The chances that two independent walks meet for a long time.
- The chances that two independent branching walks both visit a large region, and this is to be discovered during the course.

We plan to have self-contained lectures (with little pre-requisites), and will start with reviewing useful facts (classical and not) on on random walks. Here is a more detailed plan of the courses.

1. Week I: Random Walk on $\mathbb{Z}^d$ with $d \geq 3$.
    - Time spent in a a domain.
    - Green function.
    - The many forms of Capacity.
    - Covering a domain.
2. Week II: To meet or not to meet
    - Lawler's magic formula.
    - Non-intersection probabilities and Capacity.
    - Intersection of two walks for $d \geq 5$ in infinite-time horizon.
3. Week III: Branching Random Walks
    - The critical branching random walks.
    - The infinite invariant tree.
    - Green's function and Branching Capacity.
    - The critical dimension four.
    - Time spent in a ball.
4. Week IV : Branching Random Walks.
    - An approximate last passage decomposition.
    - Tail of Local Times.
    - Time spent in an arbitrary domain if $d \geq 5$.
    - Intersection of two branching random walks.

*Université Paris-Est Créteil; amine.asselah@u-pec.fr