PHD SUBJECT: AROUND UNIVERSALITY OF ROOTS OF RANDOM FUNCTIONS

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This subject aims at exploring the fine properties of sets of zeros of some specific families of random functions (random polynomials, eigenfunctions of Laplace operators,$\cdots$), with a specific emphasize on multivariate models. Typically, given a sequence of real valued deterministic functions $\{f_1, f_2, \cdots, f_n\}$ acting one some domain as well as $\{a_k\}_{k\geq 1}$ a sequence of random variables (with some prescribed distribution μ), an important object from both mathematical and physical point of view consists of

$$\mathcal{Z}_n = \left\{ x \Big| \sum_{k=1}^{n} a_k f_k(x) = 0 \right\}.$$

The asymptotic of the volume, the geometry or else the topology of $\mathcal{Z}_n$ as $n \to \infty$ are quantities of high interest when studying this kind of model. A highly non exhaustive list of references is given below, we refer to them and the references therein to get an idea of the objetcs under study. We also include at the very end a picture of the nodal sets of random eigenfunctions on the two dimensional torus.

Generally, the functions under consideration lie into finitely dimensional Euclidean spaces and picking up them randomly requires to endow these spaces with a suitable probability distribution. Due to its invariance under isometry, the standard Gaussian distribution is the most natural choice. Besides, from a purely technical point of view, Gaussianity enables to employ specific tools such as for instance, Kac-Rice formula and Malliavin calculus which are recurrent and highly efficient ingredients in the proofs of the current state of the art. Changing the probability distribution leading the choice of the random functions is self-evidently a natural question from both mathematical and physical points of view. On the one hand, one may consider more degenerate Gaussian distributions and check whether the underlaying asymptotic behaviors of nodal observables are preserved under the presence of correlations. On the other hand, one may completely remove the Gaussianity hypothesis and consider instead singular distributions. The question of universality then precisely consists in studying whether the asymptotic of the nodal observables, as the amount of noise goes to infinity, do depend on the peculiar chosen distribution or at the contrary if they coincide with the ones obtained in the Gaussian setting. The simplest example of such an universal property is the central limit theorem for renormalized sums of independent and identically distributed random variables where the limit is always a standard Gaussian, as soon as the considered random variables are centered with unit variance.

Remark 0.1. *Guillaume Poly is scientific coordinator of the ANR project "Unirandom" specifically focused on this topic, which starts on 01/01/2018 and ends on 31/12/2021. Various conferences, missions and invitations will hold during this period.*

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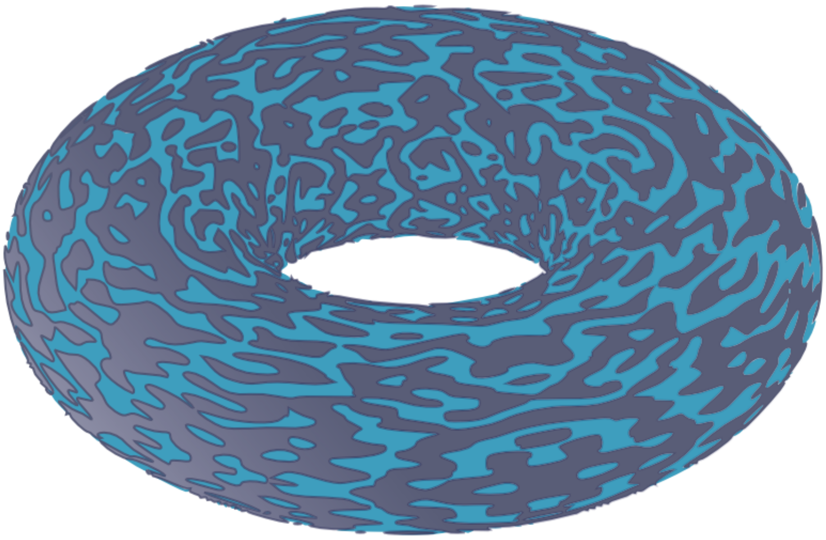

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