Normal approximation for recovery of structured unknowns: Steining the Steiner formula

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Abstract

Intrinsic volumes of convex sets are natural geometric quantities that also play important roles in applications. In particular, the discrete probability distribution $\mathcal{L}(V_C)$ given by the sequence $v_0, \ldots, v_d$ of conic intrinsic volumes of a closed convex cone C in $\mathbb{R}^d$ summarizes key information about the success of convex programs used to solve for sparse vectors, and other structured unknowns such as low rank matrices, in high dimensional regularized inverse problems. The concentration of V_C implies the existence of phase transitions for the probability of recovery of the unknown in the number of observations. Additional information about the probability of recovery success is provided by a normal approximation for V_C. Such central limit theorems can be shown by first considering the squared length G_C of the projection of a Gaussian vector on the cone C. Applying a second order Poincaré inequality, proved using Stein's method, then produces a non-asymptotic total variation bound to the normal for $\mathcal{L}(G_C)$. A conic version of the classical Steiner formula in convex geometry translates finite sample bounds and a normal limit for G_C to that for V_C.

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