Lévy processes and applications in Finance

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Syllabus

1. Lévy processes
   - Definition
   - From infinitely divisible distributions to Lévy processes
   - Lévy-Khintchine representation
   - Lévy decomposition
   - Fine structure of the Jump process
2. Classes of Lévy processes used in Finance
   - Jump Diffusion processes
   - Subordinated Brownian motions
   - An example: fitting distributions and the CGMY process
3. Option pricing
   - Itô's Lemma and the replicating portfolio - market incompleteness part I
   - Girsanov Theorem and Risk Neutral Valuation - market incompleteness part II
   - Pricing vanilla options and 'semi-closed analytical formulae'
   - PIDE
   - Applications: when does it matter? Tail events - credit risk modelling and VaR of derivatives positions
   - Shortcomings of Lévy processes and moving forward: Time Changed Lévy processes
4. Simulation and other computation issues
   - Monte Carlo simulation: plain vanilla strategy
   - Some variance reduction via stratification: bridge strategy
   - Fourier transforms

- An example: COS method

5. Linear transformation and correlated Lévy processes
    - Lévy processes in $\mathbb{R}^d$
    - Linear transformation
    - Margin processes
    - The case of the sum of independent Lévy processes
    - Correlated Lévy processes via subordination
    - Correlated Lévy processes via linear transformation
    - Applications: parameter estimation, calibration, implied correlation

The list of references provided is not exhaustive.

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