E S S A Y by Jordan Stoyanov[1]

DISTINCT AND WELL-RECOGNIZABLE NOTATIONS FOR WRITINGS IN PROBABILITY AND STATISTICS

When writing papers, books, reports and reviews, we follow good traditions and specific requirements. While in the literature, most of the used notations and abbreviations are natural, clear, appropriate and distinct, there are some, terribly misleading and tasteless.

I have discussed these issues with several well-experienced colleagues. Most of them agreed that the notations/abbreviations used in our area need to be improved and unified. However, somebody needs to start talking publicly about this, making suggestions, and then hoping that somebody else would properly act. Here is the question: ***Who and when?***

I am appealing to my *Fellows Stochasticians*, to *Publishers, Editors, Authors, Reviewers,* by referring to Lewis E. Elliott: ***If not us, then who? If not now, then when?***

Three reasonable motivating 'rules':

Rule 1. Use one letter/symbol as the notation for only one item/object.

Rule 2. For each specific group of objects, use the same font.

Rule 3. For different groups of objects use different fonts.

Good practices for fonts: 'sans serif', for probability, expectation, variance, covariance; 'mathbb', for spaces of reals, integers, complex numbers; 'mathcal', for classes, families.

Are you comfortable with the following ***terrible examples*** met in the literature?

Example 1: Let the IID $\{X_n\}$ be ID with DD ... (Meaning: IID = 'independent and identically distributed', ID = 'infinitely divisible', DD = 'dependency decreasing').

Example 2: The probability $\Pr(\xi \le x)$, its expectation $\mathbb{E}\xi$ and variance $\mathrm{var}\xi$ ... (Here 'Pr', '$\mathbb{E}$', 'var', are in one group, must be of the same font, e.g., 'sans serif'.)

Example 3: For a RV $\mathbb{X}$ on the probability space $(\Omega, \mathcal{F}, \mathbb{P})$ with values in $\mathbb{R}$, denote $\mathbb{E}[\mathbb{X}] = \int_\Omega \mathbb{X}(\omega) d\mathbb{P}$; $\mathbb{P}[\mathbb{X} \le x] = \mathbb{F}(x)$, $x \in \mathbb{R}$. (Here, totally different objects, for all used the same font 'mathbb'. Tasteless!)

Common and clear abbreviations: Some publishers do not allow abbreviations. If allowed, then use the following: i.i.d., r.v., d.f., a.s., m.g.f., cum.g.f., p.g.f., ch.f., cum.ch.f., inf.div. (no space between). The meaning of each is clear at one glance. It is old-fashionable to write RV, PDF, CDF. Density is density, why PDF? And why capitals? Why CDF, it is cumulative by definition? It is enough to write simply 'density', and 'd.f.'.

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Specific detailed list of the basics:

- For the 'standard normal density' use φ, not ϕ ('varphi' not 'phi'). 'Phi' for Φ is o'k.
- Use $\mathcal{N}(0,1)$, $\mathcal{N}(\mu,\sigma^2)$ for the normal distribution, standard and with parameters μ, σ^2.
- For spaces, sets: $\mathbb{R} = (-\infty, +\infty)$, $\mathbb{R}^n$, $\mathbb{R}_+ = [0,\infty)$, $\mathbb{C}$ = the complex plane, $\mathbb{N} = \{1,2,\ldots\}$, $\mathbb{N}_0 = \{0,1,2,\ldots\}$, $\mathbb{Z}$ or $\overline{\mathbb{N}} = \{\ldots,-2,-1,0,1,2,\ldots\}$. But not $\mathbf{R}$, etc.
- For families, σ-algebras, sets of functions, use 'mathcal': $\mathcal{A}, \mathcal{B}, \mathcal{D}, \mathcal{F}, \mathcal{C}, \mathcal{N}, \mathcal{P}, \mathcal{M}$.
- Use 'Roman font' 'e' for the Napier constant $e \approx 2.71...$
- Use 'Roman font' 'd' for differentials, not the italic d.
- For functions (= mappings), use $\mapsto$ not $\to$; the latter is 'reserved' for convergence.
- For specified, say s convergence, use $\xrightarrow{s}$, not $\to_s$ and not $\to^s$.
- For convergence, do not type $\xrightarrow{n\to\infty}$, it is accepted to write $n \to \infty$ after.
- Keep $\Gamma(\cdot)$ and $\mathrm{B}(\cdot,\cdot)$ for the classical Euler's Gamma function and Beta function.
- Use $\gamma(a,b)$, or $\mathrm{Gamma}(a,b)$, for the gamma distribution, parameters a, b.
- Use $\beta(a,b)$, or $\mathrm{Beta}(a,b)$, for the beta distribution, parameters a, b.
- Use $\mathrm{Exp}(\lambda)$ for the exponential distribution, parameter λ. But not $\exp_\lambda$.
- Use $\mathrm{Bin}(n,p)$ for the binomial distribution, parameters n, p.
- Use $\mathrm{Poi}(\lambda)$ for the Poisson distribution, parameter λ.
- Use $\mathrm{Ge}(p)$ for the geometrical distribution, parameter p.
- Keep 'mathbf' font for matrices and vectors, $\mathbf{A}, \mathbf{B}, \ldots$, $\mathbf{a}, \mathbf{b}, \ldots$.
- Use 'sans serif' font P and E for probability and expectation, not $\mathbb{P}$, $\mathbf{P}$, Pr, etc.
- For moments, use $m_k = \mathsf{E}[X^k]$, also $\mu_k = \mathsf{E}[|X|^k]$; not the tasteless $\mathcal{M}_n = \mathbb{E}[\mathfrak{X}^n]$.
- Use $\mathsf{Var}[X]$ and $\mathsf{Cov}(X,Y)$ for the variance of X and the covariance between X and Y.
- Two symbols: $X \perp Y$ for orthogonal (uncorrelated) r.v.s; $X \perp\!\!\!\perp Y$ for independent r.v.s.

Final words: There are good reasons to expect that certain things will improve the 'face' of writings in Probability and Statistics. There are masterly written books and journals of great taste also because of following most of what is above. ***Let us follow the Masters!***

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