Bivariate fractional Ornstein-Uhlenbeck process: basic properties and estimators

Motivated by the recent numerous applications of fractional Brownian motion (fBm) and fractional Ornstein Uhlenbeck (fOu) processes, notably in financial modelling, we define a correlated bivariate fractional Ornstein-Uhlenbeck process (2fOUs) $(Y_t)_{t\in\mathbb{R}} = (Y_t^1, Y_t^2)_{t\in\mathbb{R}}$, with Hurst indexes that can be different in different components.

We do so starting from the definition of multivariate fractional Brownian motions (fBms) in [ACLP13]. We consider a correlated bivariate fBm $B^{H_1,H_2} = (B^{H_1}, B^{H_2})$ and set Y^i as the fOU driven by B^{H_i}, $i = 1, 2$, where H_i is the Hurst index of the i-th component. We study the covariance structure of this process, through explicit formulas and asymptotic properties, concerning for example the long term behaviour of the covariance function, that generalize the univariate case discussed in [CKM03].

We then turn our attention to the estimation of the cross-correlation parameters. The correlation structure depends on a parameter ρ, analogous to the correlation parameter of a classical bivariate Ornstein-Uhlenbeck (or Brownian motion) process, and on an additional parameter η, ruling the time-reversibility of the processes (meaning, how $\mathbb{E}[Y_t^1 Y_s^2]$ is different from $\mathbb{E}[Y_s^1 Y_t^2]$), that appear only in the fractional case. We consider an estimator for these parameters, derived similarly to the generalized method of moments, that we show to be unbiased and consistent and, for $H_1, H_2 \in (0, 3/4)$, asymptotically Gaussian. We then consider a second estimator, based on a small time asymptotic formula for the cross-covariances $\mathbb{E}[Y_s^i Y_0^j]$, $i \neq j$, that seems more suited to estimating the correlation parameters when the mean reversion of the 2fOUs is weak, in which case some problem with identification arises. A simulation study supports these theoretical convergence results.

References

[ACLP13] P.O. Amblard, J.F. Coeurjolly, F. Lavancier, A. Philippe. *Basic properties of the multivariate fractional Brownian motion.* Self-similar processes and their applications, 63–84, Sémin. Congr., 28, Soc. Math. France, Paris, 2013.

[CKM03] P. Cheridito, H. Kawaguchi, M. Maejima. Fractional Ornstein-Uhlenbeck processes. Electron. J. Probab. 8 (2003), no. 3, 14 pp.