Stochastic Neuronal Modeling by Gauss-Markov Processes

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The theory of Gauss-Markov (GM) processes ([1]) and the First Passage Time (FPT) problem through specified boundaries turns out especially useful in the stochastic modeling of neuronal firing ([2], [3]). Pioneering models as that of Gerstein and Mandelbrot until to the widely used Leaky Integrate-and-Fire (LIF) model ([4]) and some others more recent models ([5]) are essentially based on the stochastic dynamics of Gaussian processes having the Markov property. Starting from a LIF model including time-dependent features of the neuronal dynamics, the usefulness of theoretical and numerical results related to a no-time homogeneous Ornstein-Uhlenbeck (OU) process can be shown. The use of the corresponding GM process allows to obtain reliable estimations of the neuronal firing activity and some better approximations of results as those highlighted, for instance, in [6]. Then, the need of describing several phenomena, such as interactions between neurons ([7]), effects of input currents ([8]), adaptation of the firing activity ([9],[10]), occurrence of spike trains ([11], [12]) have led us to design specialized neuronal models and consequently to construct suitable GM processes. How this approach can be useful to predict some aspects of the neuronal firing activity is also shown by comparing numerical and simulation results.

References

[1] E. Di Nardo, A.G. Nobile, E. Pirozzi and L.M. Ricciardi, A computational approach to first passage-time problems for Gauss-Markov processes. *Adv. Appl. Prob.*, (2001) 33:453–482.

[2] A. Buonocore, L. Caputo, E. Pirozzi and L.M. Ricciardi, On a Stochastic Leaky Integrate-and-Fire Neuronal Model, *Neural Computation*, **22**, 2558–2585, (2010).

[3] A. Buonocore, L. Caputo, E. Pirozzi and L.M. Ricciardi, The first passage time problem for gauss-diffusion processes: Algorithmic approaches and applications to lif neuronal model. *Methodol. Comput. Appl. Probab.*, 13:29-57, (2011).

[4] A.N. Burkitt, A review of the integrate-and-fire neuron model: I. Homogeneous synaptic input, *Biological Cybernetics*, **95** (2006), 1–19.

[5] L. Sacerdote , M. T. Giraudo, Stochastic Integrate and Fire Models: A Review on Mathematical Methods and Their Applications. In Stochastic Biomathematical Models, Volume 2058 of *Lecture Notes in Mathematics*, (2012) pp 99-148.

[6] S. Shinomoto, Y. Sakai and S. Funahashi, The Ornstein-Uhlenbeck process does not reproduce spiking statistics of cortical neurons, *Neural Computation*, **11** (1997), 935–951.

[7] A. Buonocore, L. Caputo, E. Pirozzi and M.F. Carfora, Gauss-diffusion processes for modeling the dynamics of a couple of interacting neurons, *Math. Biosci. Eng.*, **11** (2014), 189–201.

[8] M. F. Carfora, E. Pirozzi. Stochastic modeling of the firing activity of coupled neurons periodically driven. In: *Dynamical Systems and Differential Equations*, AIMS Proceedings 2015. p. 195-203, AIMS, America Institute of Mathematical Science, Madrid (Spain), (2015).

[9] H. Kim and S. Shinomoto, Estimating nonstationary inputs from a single spike train based on a neuron model with adaptation, *Math. Bios. Eng.*, **11** (2014), 49-62.

[10] A. Buonocore, L. Caputo, E. Pirozzi and M.F. Carfora. A Leaky Integrate-And-Fire Model With Adaptation For The Generation Of A Spike Train. *Mathematical Biosciences and Engineering* Volume 13, Number 3, In Print.

[11] G. D'Onofrio, E. Pirozzi, Successive Spike Times Predicted By A Stochastic Neuronal Model With A Variable Input Signal *Mathematical Biosciences and Engineering* Volume 13, Number 3, In Print.

[12] G. D'Onofrio, E. Pirozzi, M.O. Magnasco. Towards Stochastic Modeling of Neuronal Interspike Intervals Including a Time-Varying Input Signal. In: Computer Aided Systems Theory EUROCAST 2015. LNCS vol. 9520, p. 166-173, Springer, (2015).