DISSIPATIVE SQG EQUATIONS DRIVEN BY SPACE-TIME WHITE NOISE

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Abstract: We consider the dissipative surface quasi-geostrophic (SQG) equation driven by space-time white noise ξ in the two-dimensional torus, which is formally given by

$$\begin{cases} \partial_t\theta + (-\Delta)^\gamma\theta &= u\cdot\nabla\theta + \xi, \\ u &= \nabla^\perp(-\Delta)^{-1/2}\theta, \end{cases}$$

where $\gamma \in (0, 1]$ and $\nabla^\perp = (\partial_{x_2}, \partial_{x_1})$.
The SQG equation describes (roughly speaking) the temperature θ in a rapidly rotating stratified fluid which is transported by the velocity field u and the term $(-\Delta)^\gamma\theta$ is due to Ekman pumping. The operator $\nabla^\perp(-\Delta)^{-1/2}$ connecting the velocity field to the temperature can be rewritten as a Riesz-transform, which are singular integral operators. The equation has applications in both meteorological and oceanic flows, while in mathematics it is often used as a toy model for the 3D Navier-Stokes equations due to some structural similarities of these equations. We will give a brief overview on the results for different versions of stochastic SQG equations and then turn to the question of the existence local solutions to the SQG equation driven by space-time white noise. We will use the theory of regularity structures and show how the special structure of the Riesz transform allows us to lift such an operator to the spaces of modelled distributions. Using this result we then sketch the existence proof and give some details on the renormalisation procedure.