Avviso di Seminario

A sharp bound on the expected number of upcrossings of an L_2-bounded Martingale*

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Consider a martingale $(M_n)_{n\geq 0}$, starting at x. It is well known that for any martingale it holds $\mathbb{E}[M_n] = \mathbb{E}[M_0] = x$. Now, let M be L_2- bounded and assume that the final coordinate or limit Y be such that $Var(Y) = \sigma^2$. We investigate how much variability can M have, allowed by σ^2. In the past Dubins and Schwarz (1988) proved that $\mathbb{E}[\max(M)] = \sigma$ and $\mathbb{E}[\max|M|] = \sigma\sqrt{2}$, while Dubins, Gilat and Meilijson (2009) showed that $\mathbb{E}[\max(M) - \min(m)] = \sigma\sqrt{3}$.

Here, we give another bound to the variability of the considered martingale in terms of the expected number of up-crossings of an interval. In particular, we prove that the upper bound for the expected number of up- crossings of (a, b) by M is $\sigma/2$ and that this bound is attained by a martingale starting at $x = a$. To prove this result we use the Doob's up-crossing inequality.

In our approach we denote with $\Delta = (b - a)/\sigma$ and $\delta = |x - a|/\sigma$ the normalized length of the interval and of the distance from the initial point to the lower endpoint, respectively. Then we prove that the expected number of up-crossings of (a, b) by M is at most $\frac{\sqrt{1+\delta^2}-\delta}{2\Delta}$ if $\Delta^2 \leq 1 + \delta^2$ and at most $\frac{1}{1-(\Delta+\delta)^2}$ otherwise. Both bounds are sharp, attained by a Standard Brownian Motion stopped at appropriate times. Furthermore, we show that both bounds attain the Doob's upper bound on the expected number of up-crossings of (a, b).

*Joint work with David Gilat and Isaac Meilijson (Department of Statistics and Operations Reseach, Tel Aviv University)

Il seminario si terrà il giorno 27 Aprile 2017 ore 12:00 nella Sala Professori secondo livello del Dipartimento Matematica e Applicazioni, Università di Napoli FEDERICO II, Complesso di Monte Sant'Angelo, Via Cintia, Napoli.