Curriculum Vitae of Tamara Servi

Personal information

- Name : Tamara Servi
- Date of birth : 29/12/1976
- Nationality : Italian.
- Postal address : CMAF Universidade de Lisboa, Av. Prof. Gama Pinto 2, 1649-003, Lisboa (Portugal)
- e-mail address : tamara.servi@googlemail.com
- Web page : http://ptmat.fc.ul.pt/tservi/
- Current position: post-doc researcher at CMAF Universidade de Lisboa.

Research domain

O-minimal geometry. Model Theory applied to Real Geometry and Number Theory. Resolution of singularities for quasi-analytic algebras. Effective methods in Real Geometry and decidability problems.

University degrees

Year	**Degree**	**Institution**	**Mark**
2007	PhD in Mathematics	Scuola Normal Superiore di Pisa	70/70 cum laude
2001	Laurea in Matematica	Università di Pisa	110/110 cum laude

Situation et professional experience

Period	**Position**	**Institution**
July 2008 -	Post-doc researcher	CMAF Universidade de Lisboa (Portugal)
March 2008 - May 2008	Invited researcher	Université de Mons (Belgium)
March 2006 - February 2008	Teaching assistant	Universität Regensburg (Germany)
January 2005- February 2006	Pre-doc researcher (European Grant RAAG)	Universität Passau (Germany)

Prizes and grants

Year	**Title**	**Institution**
2008	Best PhD Thesis	AILA - associazione italana di logica e applicazioni
2005	Research Grant (6 months)	PRIN Modelli e Insiemi
1996	First prize as a talented student	Fondazione privata dipendente dall' Accademia dei Lincei

Languages spoken

Italian (native speaker); English (excellent); French (very good); German (good); Portuguese (good).

Organisation of scientific events and participation in research projects

December 2005	Organisation of the Workshop on Model Theory, Real Geometry and Optimization (Konstanz, Germany).
2005-2006	Organisation of a seminar for students and young researchers (Regensburg, Germany).
February 2009	Joint management of a 2-year project on o-minimality involving the Logic Group at CMAF and other international researchers, financed by the FCT (Foundation for science and technology in Portugal)
October 2009	Organisation of the Workshop on the Model Theory of Power Functions (Lisboa, Portugal). http://www.ciul.ul.pt/ logicmat/lgwebwrpf.
November 2009	Joint management of a 1-year project in model theory involving the Logic Group at CMAF and the members of the Logic Group at the university of Lyon I, financed jointly by the FCT and the CNRS
September 2010	Joint management of a 1-year project on o-minimality involving the Logic Group at CMAF and the members of the Geometry Group at the university of Dijon, financed jointly by the FCT and the CNRS
February 2011	Joint management of a 2-year project on o-minimality involving the Logic Group at CMAF and other international researchers, financed by the FCT (Foundation for science and technology in Portugal)
February 2012	Organisation of the Workshop on o-minimal preparation theorems (Lisboa, Portugal). http://www.ciul.ul.pt/ logicmat/lgwebwopt.html.

Publications

Thesis

Tamara Servi."On the First-Order Theory of Real Exponentiation", Pisa, Edizioni della Normale, 2008, ISBN 978-88-7642-325-3.

My thesis has been selected by a committee of referees for publication in the series "Theses" of the Edizioni della Scuola Normale Superiore di Pisa.

Accepted and published papers

- (with A. Berarducci), "An effective version of Wilkie's theorem of the complement and some effective o-minimality results", Annals of Pure Applied Logic 125 (2004), no. 1-3, 43–74.
- (alone), "Noetherian varieties in definably complete structures" Logic and Analysis, Vol. 1, No. 3. (1 November 2008), pp. 187-204.
- (with A. Fornasiero), "Definably Complete Baire Structures", Fund. Math. 209 (2010), no. 3, 215–241.
- (with A. Fornasiero), "Theorems of the Complement", to appear in "Proceedings of the Thematic Semester in o-minimality at the Fields Institute of Toronto (Spring 2009)".
- (with A. Fornasiero), "Pfaffian closure for Definably Complete Baire Structures", to appear in Illinois Journal of Mathematics.
- (with G. Jones), "On the Decidability of the Real field with a Generic Power Function", Journal of Symbolic Logic 76 (2011), no. 4, 1418–1428.

Preprints

- (with Jean-Philippe Rolin), "Rectilinearisation theorem for generalised quasi-analytic algebras".

Teaching

Year	Course	Role	Institution
2003	Discrete Mathematics	Teaching assistant	Univ. of Pisa
2006	Linear Algebra	Teaching assistant	Univ. of Regensburg
2006	Algebra	Teaching assistant	Univ. of Regensburg
2007	Logic	Teaching assistant	Univ. of Regensburg
2007	Linear Algebra	Teaching assistant	Univ. of Regensburg
2008	Geometry for school teachers	Seminar	Univ. of Regensburg

I have taught in Italian, English and German. My research contract in Portugal is not compatible with doing any teaching.

Invited talks

University of Regensburg (Germany), November 2003	in the real geometry seminar.
University of Notre Dame, Indiana (U.S.A.), April 2004	in the logic seminar.
University of Illinois at Urbana-Champaign (U.S.A.), April 2004	in the logic seminar.
Wesleyan University, Connecticut (U.S.A.), April 2004	in the logic seminar.
University of Torino (Italy), July 2004	in the Logic Colloquium 2004.
University of Torino (Italy), April 2007	in the conference PRIN Modelli e Insiemi.
University of Perugia (Italy), June 2007	in the joint meeting UMI-DMV (Italian and German mathematical societies).
Université de Paris 7 (France), April 2008	in the logic seminar.
University of Barcelona (Spain), November 2008	in the final MODNET conference (European Network of Model Theory).
University of Manchester (U.K.), December 2008	in the logic seminar.
Fields Institute, Toronto (Canada), April 2009	in the seminar of the special semester in o-minimality.
Fields Institute, Toronto (Canada), June 2009	in the Workshop on Decidability.
Stefan Banach Mathematical Research and Conference Center, Bedlewo (Poland), August 2009	in the ESF Model Theory Meeting.
Université de Mons (Belgium), December 2009	in the Anglo-Belgian Workshop in Model Theory.
Universités Paris 6 et 7 (France), December 2009	in the seminar Structures Algébriques Ordonnées.
University of Münster (Germany), January 2010	in the logic seminar.
Université de Lyon (France), February 2010	in the logic seminar.
Université de Bourgogne (France), February 2011	in the geometry seminar.
Université de Nice (France), May 2011	in the meeting Singularités Réelles et Systèmes Dynamiques.
ENS de Paris (France), June 2011	in the seminar GTM Géométrie et Théorie des modèles.
Université de Rennes, June 2011	in the Real Algebraic Geometry Conference.
Fields Institute, Toronto (Canada), August 2011	in the o-minimal Structures and Real Analytic Geometry Retrospective Workshop.
Université de Bourgogne (France), January 2012	in the geometry seminar.
Université de Chambéry (France), January 2012	in the geometry seminar.
Université de Rennes (France), January 2012	in the geometry seminar.
Université de Bordeaux (France), March 2012	in the geometry seminar.
Université de Marseille (France), April 2012	in the geometry seminar.
Max Planck Institut für Mathematik, Bonn (Germany), June 2012	in the final workshop of the trimester in Model Theory
Université de Paris 6 (France), June 2012	in the summer school "Around the Zilber-Pink conjectures"

Research interests and competences

My main center of interest is *o-minimal geometry.* The notion of *o-minimal structure* was introduced by Lou van den Dries (1984) in connection to the problem, due to Tarski (1951), of the decidability of the real exponential field. The framework of o-minimal geometry, which includes that of semi-algebraic and semi-analytic geometry, can be seen as a candidate for Grothendieck's "tame topology" programme, described in his "Esquisse d'un Programme" (1984). It essentially consists in studying the topological properties of a family of sets obtained by some natural constructions starting from certain real functions; one requires that such a family be rich enough to contain interesting and nontrivial constructions, but at the same time one stipulates that such a family of sets do not generate any pathological phenomena (for example one can associate a reasonable notion of dimension to such sets, and no oscillation phenomena are allowed).

My main competences lie in the interactions between Model Theory, o-minimal Geometry and Number Theory. If we consider the sub-analytic sets defined by certain classical functions which satisfy algebraic differential equations (for example the exponential function, the trigonometric functions, the Weierstrass $\wp$-functions), we realise that the deep number theoretic properties of such functions determine the geometry of the sets they define. Many studies, due for example to Macintyre and Wilkie (1996), Bianconi (1997), Pila and Wilkie (2006), have shown that model theory often provides a link between number theoretic and geometric methods.

Results obtained

Tarski (1951) proved that the projection $\pi(A)$ of a semi-algebraic set A is also semi-algebraic (this result is known as *quantifier elimination*). Moreover, there is an effective way to find polynomial equations and inequalities describing $\pi(A)$ from the polynomial equations and inequalities which describe A. This reflects the fact that the theory of the real field is *decidable*: there is an algorithm which decides the truth of all elementary sentences (involving solely sums and products of real numbers).

Tarski asked whether his decidability results could be generalised to more complicated structures, for example the real field with the exponential function $\mathbb{R}_{\exp}$. This question is linked to profound issues in transcendental number theory: for example, if the theory of $\mathbb{R}_{\exp}$ were decidable, then one could establish, given a polynomial $p(x,y,z) \in \mathbb{Z}[x,y,z]$, if $p(e, e^e, e^{e^e}) = 0$ (we remind that it is not known whether these three numbers are algebraically independent).

The following famous conjecture, due to Schanuel, resumes this kind of questions: if $a_1, \ldots, a_n \in \mathbb{R}$ are $\mathbb{Q}$-linearly independent, then the transcendence degree over $\mathbb{Q}$ of the field $\mathbb{Q}(a_1, \ldots, a_n, e^{a_1}, \ldots, e^{a_n})$ is at least n. Assuming this conjecture (which is generally considered as out of reach for the moment), Macintyre and Wilkie (1996) proved that the theory of $\mathbb{R}_{\exp}$ is decidable. It is thus natural to ask what kind of results one could obtain in this direction without assuming any unproven conjectures.

In my thesis I answer unconditionally a whole class of decidability questions for the theory of $\mathbb{R}_{\exp}$.

Theorem (Thesis). *There exist effective bounds on the number of connected components of the sets definable in* $\mathbb{R}_{\exp}$.

In my subsequent work with A. Fornasiero, we generalised this result to a larger class of structures, using different methods (by proving some uniform finiteness results for a class of nonarchimedean structures and then using model theoretic compactness).

In my work with G. Jones, we give a more explicit and complete answer to an instance of Tarski's problem, thus exhibiting the first example of a proper expansion by functions of the real field whose theory is decidable.

Theorem (with G. Jones, 2009). *Let $\alpha \in \mathbb{R}$ be computable and "exponentially transcendental" (roughly, α does not appear as a coordinate of a zero of a system of exponential polynomial equations. Then the theory of the real filed with the power function x^α is decidable.*

There are two sides to Tarski's theorem, the decidability result, which has been discussed above, and the quantifier elimination result. One could ask if the quantifier elimination result could be generalised to more complicated structures, such as the expansion of the real field by some family of real analytic function. However, it is well known that the projection of a semi-analytic set is not in general semi-analytic (this is why one introduces the category of sub-analytic sets). Nevertheless, Denef and van den Dries (1988) proved that every relatively compact sub-analytic set can be described by a system of equations and inequalities satisfied by some compositions of analytic functions and quotients (i.e. the theory of the expansion of the real field by all restricted analytic functions eliminates quantifiers in the language expanded by a symbol for division).

In my work with J.-P. Rolin we consider a family of functions which generalises the class of real analytic functions. We consider a family of algebras of real functions such that the germ at zero of each function has an "asymptotic development" which is a formal power series with natural of positive real exponents. We require the following *quasi-analyticity* property: the germ of each function in the class is "characterised" by its asymptotic development, in the sense that the algebra morphism which associates to every germ its asymptotic development is *injective*. There are many examples, coming from problems in dynamical systems, where the study of such algebras of functions arises naturally, namely the unsolved issues around Dulac's problem.

One of the key ingredients of Denef and van den Dries' proof is the Weierstrass Preparation Theorem, which is known to fail in the quasi-analytic setting. However, by different methods and using o-minimality, we are able to prove the following statement.

Theorem (with J.-P. Rolin, 2012). *The theory of the expansion of the real field by a family of quasi-analytic algebras eliminates quantifiers in the language expanded by a symbol for division.*

Current and future directions

All known polynomially bounded o-minimal expansion of the real field are generated by a family of quasi-analytic algebras and hence satisfy the hypotheses of the above theorem. Hence it is natural to ask if a similar construction could be done in the non-polynomially bounded case. In particular, certain functions appearing in questions related to Dulac's problem have an asymptotic expansion in the form of a formal series with logarithmic-exponential monomials and Ilyashenko (1991) proved that the asymptotic expansion determines the germ of the function uniquely, in the sense of quasi-analyticity. Therefore, I propose to generalise the setting of the above theorem to the log-exponential case and try to reproduce the same scheme of proof. A preliminary work is required in order to give an appropriate definition of log-exp series in several variables and to define accordingly a generalised notion of quasi-analyticity.

One of the intermediate steps in the proof of the above theorem is to establish a covering theorem for the sets definable in the structures under consideration, which essentially states that every definable set can be written as a finite union of diffeomorphic images of quadrants via maps whose components are in the quasi-analytic algebras which generate the structure under consideration.

Together with G. Jones and M. Thomas we aim to use the above covering theorem to investigate the existence of *mild parameterisations* for some polynomially bounded o-minimal structures, such as a structure generated by a class of Gevrey functions, introduced by van den Dries and Speissegger (1999). Mild parameterisations play a key role in approaching questions arising from the interaction of diophantine geometry and model theory and have been used for example by Pila and Wilkie (2006) in establishing a result on the density of rational points on sets definable in o-minimal structures. Wilkie's conjecture states that, for sets definable in $\mathbb{R}_{\exp}$, this bound can be improved and is of the form "power of log". In investigating Wilkie's conjecture it would be useful to have an analogue to the covering theorem in the non-polynomially bounded case, in order to try to obtain mild parameterisations for sets in this structure.

In the same direction, together with G. Jones and J. Kirby we plan to investigate the following interdefinability problem: let f be a transcendental analytic function which is locally definable from a family of Weierstrass $\wp$-functions; can f be definable in $\mathbb{R}_{\exp}$? We expect a negative answer, which could be used to obtain power-of-log bounds for the density of the points of the form $(\log a, \log b)$, for $a, b \in \mathbb{Q}$, lying on the graph of f. Pila (2010) uses this type of bounds to obtain a weak version of The Six Exponentials Theorem, and our result could be used to obtain its analogue for elliptic curves.