Curriculum vitae

Gabriele Ranieri

Personal

Date of birth	April 11, 1979
Birthplace	Lucca (Italy)
Citizenship	Italian
Marital status	Married
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Past and current positions

September 1, 2007 - August 31, 2008 A.T.E.R. University of Caen, France.
September 1, 2008 - August 31, 2009 A.T.E.R. University of Caen, France.
September 1, 2009 - August 31, 2010 Post-doc University of Basel, Switzerland.
October 1, 2010 - March 31, 2011 Post-doc University of Goettingen, Germany.
April 1, 2011 - May 31, 2011 Invitation at Max Planck Intitüt, Bonn, Germany.
October 1, 2011 - September 30, 2012 Junior visiting position at Centro De Giorgi, Pisa, Italy.

University cursus

2007	P.H.D. thesis. Mention: très honorable (very good).
2004-2007	P.H.D. in mathematics at Pisa University (Italy). Since 2005 P.H.D. in mathematics at Caen University (France) and “Cotutelle” between the universities of Pisa and Caen.
2003	Laurea (master’s degree) at Pisa University (Italy). Mention: 110/110 cum laude.
1998-2003	Student at Pisa University.

1 Research

1.1 Thesis

Original title	Rang de l'image du groupe des unités; conjecture de Bremner.
English title	Rank of the image of the units group; Bremner's conjecture.
Thesis advisors	Francesco AMOROSO (University of Caen), Roberto DVORNICICH (University of Pisa)

1.2 Articles

1. G. RANIERI, *Rang de l'image du groupe des unités et polynômes lacunaires*, Acta Arithmetica **127**, 87-96 (2007). (Rank of the image of units group and lacunary polynomials).

   ABSTRACT. Amoroso [Amo] introduced a new class of number fields, the so-called class of PCM-fields (proche d'un corps CM), having interesting arithmetic properties. The main result of our work is the proof of the fact that no totally real field is PCM. Moreover, we show how our proof, in the special case of finite totally real abelian extensions of $\mathbb{Q}$, implies a lower bound for the number of non-zero coefficients of polynomials, whose certain zeros are roots of 1.

2. G. RANIERI, *Générateurs de l'anneau des entiers d'une extension cyclotomique*, Journal of Number Theory **128**, 1576-1586 (2008). (Power integral bases of a cyclotomic extension).

   ABSTRACT. Let p be an odd prime and $q = p^m$, where m is a positive integer. Let ζ_q be a primitive qth root of 1 and $\mathcal{O}_q$ be the ring of integers of $\mathbb{Q}(\zeta_q)$. In [Ga-Ro], I. Gaál and L. Robertson showed that if $(h_q^+, p(p-1)/2) = 1$ (where h_q^+ is the class number of $\mathbb{Q}(\zeta_q + \overline{\zeta_q})$) then, if $\alpha \in \mathcal{O}_q$ is a generator of $\mathcal{O}_q$ either α is equal to a conjugate of an integer translate of ζ_q or $\alpha + \overline{\alpha}$ is an odd integer. In this paper we show that we can remove the hypothesis over h_q^+. In other words we prove that if α is a generator of $\mathcal{O}_q$, then either α is a conjugate of an integer translate of ζ_q or $\alpha + \overline{\alpha}$ is an odd integer.

3. G. RANIERI, *Power bases for rings of integers of abelian imaginary fields*, Journal of the London Math. Soc. **82**, 144-166 (2010).

   ABSTRACT. Let L be a number field and let $\mathcal{O}_L$ be its ring of integers. It is a very difficult problem to decide whether $\mathcal{O}_L$ has a power basis. Moreover, if a power basis exists, it is hard to find all the generators of $\mathcal{O}_L$ over $\mathbb{Z}$. In this paper, we show that if α is a generator of the ring of integers of an abelian imaginary field whose conductor is relatively prime to 6, then either α is an integer translate of a root of unity, or $\alpha + \overline{\alpha}$ is an odd integer. From this result and other remarks it follows that if β is a generator of the ring of integers of an abelian imaginary field with conductor relatively prime to 6 and β is not an integer translate of a root of unity, then $\beta\overline{\beta}$ is a generator of the ring of integers of the maximal real field contained in $\mathbb{Q}(\beta)$.

   Finally, if $d > 1$ is an integer relatively prime to 6, we prove, using the main result of Gras [Gra], that all but finitely many abelian imaginary extensions of $\mathbb{Q}$ of degree $2d$ have a ring of integers that does not have a power basis.

4. B. ANGLÈS, G. RANIERI, *On the linear independence of p-adic L functions modulo p*, Annales de l'Institut Fourier **60**, no. 5, 1831-1855, (2010).

   ABSTRACT. Let $p \geq 3$ be a prime. Let $n \in \mathbb{N}$ be such that $n \geq 1$, let $\chi_1, \dots, \chi_n$ be characters of conductor d not divisible by p and let ω be the Teichmüller character. For all i between 1 and n, for all j such that $0 \leq j \leq (p-3)/2$, set

$$\theta_{i,j} = \begin{cases} \chi_i \omega^{2j+1} & \text{if } \chi_i \text{ is odd;} \\ \chi_i \omega^{2j} & \text{if } \chi_i \text{ is even.} \end{cases}$$

Let K be the smallest extension of $\mathbb{Q}_p$ that contains all the values of the characters χ_i and let π be a prime of the valuation ring $\mathcal{O}_K$ of K. For all i, j let $f(T, \theta_{i,j})$ be the Iwasawa series associated to $\theta_{i,j}$ and $\overline{f(T, \theta_{i,j})}$ its reduction modulo (π). Finally let $\overline{\mathbb{F}_p}$ be an algebraic closure of $\mathbb{F}_p$. Our main result is that if the characters χ_i are all distinct modulo (π), then 1 and the series $\overline{f(T, \theta_{i,j})}$, for $1 \leq i \leq n$ and $0 \leq j \leq (p-3)/2$, are linearly independent over a certain field Ω that contains $\overline{\mathbb{F}_p}(T)$. In the case $d = 1$, the linear independence of the series $\overline{f(T, \theta_{i,j})}$ seems to confirm the hypothesis that Ferrero and Washington made to build an heuristic to determine a bound for the λ-invariant of $\mathbb{Q}(\zeta_p)$.

L. PALADINO, G. RANIERI, E. VIADA, *On local-global divisibility by p^n in elliptic curves*, accepted on the Bulletin of the London Mathematical Society, arXiv:1104.4762.

ABSTRACT Let p be a prime number and let k be a number field not containing $\mathbb{Q}(\zeta_p + \overline{\zeta_p})$. Let $\mathcal{E}$ be an elliptic curve defined over k. We show that, if there exists a counter-example to the local global-divisibility problem by p^n over $\mathcal{E}$, then $\mathcal{E}$ admits a k-rational ponit of order p. By some deep results of Merel [Mer], Mazur [Maz] and Kamienny [Kam], we get that for every prime number p greater than a constant $C([k : \mathbb{Q}])$, depending on $[k : \mathbb{Q}]$, for every elliptic curve $\mathcal{E}$ defined over k, there does not exist counter-examples to the local-global divisibility by p^n over $\mathcal{E}$. In particular, $C(1) \leq 7$ and $C(2) \leq 13$.

L. PALADINO, G. RANIERI, E. VIADA, *On the minimal set for counterexamples to the local-global principle* , Preprint, arXiv:1107.3431, submitted.

ABSTRACT Let p be a prime number and let k be a number field not containing $\mathbb{Q}(\zeta_p + \overline{\zeta_p})$. Let $\mathcal{E}$ be an elliptic curve defined over k. We show that, if there exists a counter-example to the local-global divisibility problem by p^n over $\mathcal{E}$, then $\mathcal{E}$ has a k-rational point of order p, $k(\mathcal{E}[p]) = k(\zeta_p)$ and there exists a cyclic k-isogeny of degree p^3 between two elliptic curves defined over k and k-isogenous to $\mathcal{E}$. By using this criterion and a result of Kenku (see [Ken]), we get that for every prime number $p \geq 5$, for every elliptic curve $\mathcal{E}$ defined over $\mathbb{Q}$, there does not exist a counter-example to the local-global divisibility by p^n over $\mathcal{E}$.

2 Teaching experience

1. TD Approfondissement en mathématiques – September 2007- January 2008 – University of Caen.
2. TD L1 Analyse – February-May 2008 – University of Caen.
3. TD L1 Algèbre – February-May 2008 – University of Caen.
4. Cours-TD L1 Analyse – February-May 2009 – University of Caen.
5. TD Master Galois Theory – September 2009-January 2010 – University of Basel.
6. TD Master Commutative Algebra – March-June 2010 – University of Basel.

3 Talks and conferences

1. *Conference Sa conjectura de Catalan* – Menorca – September 2004.
   Talk: Plus argument semisimple.
2. *Trimestre di geometria diofantea* – Pisa (Scuola Normale) – March-June 2005.
3. *XVII^e rencontres arithmétiques de Caen* –« Approximation diophantienne » –June 2006.

4. *Colloque Approximation diophantienne et nombres trascendants* – Luminy, Marseille (C.I.R.M.) – September 2006.
   Talk: Une minoration du nombre de coefficients non nuls de polynômes s'annullant en certaines racines de l'unité.

5. *Seminaires du groupe de théorie des nombres de Caen* – Caen (L.M.N.O.) – October 13, 2006.
   Talk: Générateurs de l'anneau des entiers d'une extension cyclotomique.

6. *Workshop Diophantische Approximationen* – Oberwolfach (M.F.O.) – April 2007.
   Talk: Power integral bases for prime-power cyclotomic fields.

7. *Workshop Diophantine equations* – Leiden (Lorentz Center) – May 2007.

8. *Journées arithmétiques* – Edimbourgh – July 2007.
   Talk: Power integral bases for prime-power cyclotomic fields.

9. *Seminaires du groupe de théorie des nombres de Bordeaux* – Bordeaux – November 9, 2007.
   Talk: Générateurs de l'anneau des entiers d'une extension cyclotomique.

10. *XIX^e rencontres arithmétiques de Caen* –« Arithmétique des corps de fonctions en caractéristique positive » – June 2008.

11. *Iwasawa 2008* – Kloster Irsee – June-July 2008.

12. *Seminaires du groupe de théorie des nombres de Besançon* – Besançon – March 12, 2009.
    Talk: Indépendence linéaire des fonctions L p-adiques modulo p.

13. *XX^e rencontres arithmétiques de Caen* –« Cohomologie p-adique » – June 2009.

14. *Journées arithmétiques* – Saint-Etienne – July 2009.
    Talk: Power integral bases for abelian imaginary fields.

15. *Number theory seminar, Basel* – Basel – October, 8, 2009.
    Talk: On the linear independence of p-adic L-functions modulo p.

16. *Séminaires du groupe de travail p-adique Université de Caen* – Caen (L.M.N.O.) – February, 12, 2010.
    Talk: Méthodes diophantiennes en théorie d'Iwasawa.

17. *Séminaires du groupe de théorie des nombres de Bordeaux* – Bordeaux – May, 28, 2010.
    Talk: Indépendence linéaire des fonctions L p-adiques modulo p.

18. *Seminario di algebra dell'Università della Calabria* – Cosenza – October 2010.
    Talk: Indipendenza lineare delle funzioni L p-adiche, modulo p.

19. *Séminaires du groupe de théorie des nombres de Caen* – Caen (L.M.N.O.) – March, 11, 2011.
    Talk: Divisibilité locale-globale dans les courbes elliptiques.

20. *Groupe De Travail de Géométrie Diophantienne de Bordeaux* – Bordeaux – May, 5, 2011.
    Talk: Divisibilité locale-globale dans les courbes elliptiques.

21. *Journées arithmétiques* – Vilnius – June-July 2011.

22. *Séminaires du groupe de théorie des nombres de Caen* – Caen (L.M.N.O.) – December, 9, 2011.
    Talk: Sur la divisibilité locale-globale sur les courbes elliptiques.

23. *Seminars on Scuola Normale Superiore Pisa* – Pisa (Scuola Normale) – February, 2012.
    Talk: La congettura di Serre nel caso split-Cartan, dopo Bilu e Parent.

References

[Amo] FRANCESCO AMOROSO, *Groupes des classes de corps proches d'un corps CM*, Preprint (2005).

[Ga-Ro] ISTVÀN GAÁL, LEANNE ROBERTSON, *Power integral bases in prime-power cyclotomic fields*, J. of Number Theory, **120**, 372-384, (2006).

[Gra] MARIE NICOLE GRAS, *Non monogénéité de l'anneau des entiers de certaines extensions abéliennes de* $\mathbb{Q}$, Publ. Math. Sci. Besançon, Théor. Nombres, (1984).

[Kam] KAMIENNY, S., *Torsion points of elliptic curves and q-coefficients of modular forms*, Invent. Math., no. **109**, 221-229, (1992).

[Ken] KENKU M. A., *On the modular curves $X_0(125), X_1(25)$ and $X_1(49)$*, J. London Math. Soc. (2) **23**, no. 3, (1981).

[Maz] MAZUR B., *Rational isogenies of prime degree (with an appendix of D. Goldfeld)*, Invent. Math., no. **44**, 129-162, (1978).

[Mer] MEREL L., *Bornes pour la torsion des courbes elliptiques sur les corps de nombres*, Invent. Math. **124** no. 1-3, 437-449, (1996).